Discrete-Time LQ Stochastic Control with Equality-Constrained Inputs: Application to Energy Demand Response

Léo Seugnet and Shuang Gao

Department of EE, Polytechnique Montreal & GERAD

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Outline

Motivation

Electric vehicle (EV) integration

Solar generation

Wind generation

- Challenge: modern power grids under renewable transition face growing uncertainties due to large-scale electric vehicle adoption, variable solar generation, and intermittent wind generation
- Key question: how to coordinate user populations to meet the supply-demand balance?

Scalable collaboration with equality constraints to bring certainty!

Literature Review

Renewable energy integration and demand response.

- Storage is a key tool to mitigate uncertainty, reduce peak consumption, and manage generation loss (e.g. Tesla Virtual Power Plant, Hilo) [Kataray 2023]; [Pallage 2024].
- Deterministic LQ formulation of a demand-response problem [Le Floch 2015].

Deterministic LQ control with constraints.

- Inequality constraints on state and/or control [Scokaert 1998]; [Chmielewski 1998]; [Mare 2007]; [Chang 2013].
- Equality constraints on state and/or control [Ko 2007]; [Sideris 2010]; [Laine 2019]; [Laurenzi 2025].

Stochastic LQ control with constraints.

- Inequality constraints on state and/or control [Hassan 2016]; [Chen 2016]; [Wu 2020].
- Equality constraints on the state [Krokavec 2008], [2011], [2014].

	No constraint	**Inequality**	**Equality**
Deterministic	classical LQR	[Scokaert]; [Mare]; [Chang]	[Ko]; [Sideris]; [Laine]; [Laurenzi]
Stochastic	classical LQG	[Hassan]; [Chen]; [Wu]	state: [Krokavec]; **control: this work**

System Model

Consider N heterogenous collaborative users (e.g. consumers with batteries) over a finite horizon $[\![0, T]\!]$.

Agent i dynamics

$$x_{i,t+1} = A_i x_{i,t} + B_i u_{i,t} + w_{i,t}, \tag{1}$$

where $\{w_{i,t}\}$ are i.i.d. Gaussian processes with mean zero.

Let $x_t = [x_{1,t}, \cdots, x_{N,t}]^\intercal$, $u_t = [u_{1,t}, \cdots, u_{N,t}]^\intercal$ and $w_t = [w_{1,t}, \cdots, w_{N,t}]^\intercal$. The compact representation of the N-agent system is given by

Global system dynamics

$$x_{t+1} = Ax_t + Bu_t + w_t, \tag{2}$$

where $A = \mathrm{diag}(A_1, \ldots, A_N)$, $B = \mathrm{diag}(B_1, \ldots, B_N)$.

System Objective

Agent i instantaneous cost at time t

$$\ell_i(x_{i,t}, u_{i,t}) = (x_{i,t} - r_{i,t})^\intercal Q_i (x_{i,t} - r_{i,t}) + (u_{i,t})^\intercal R_i (u_{i,t}), \quad Q_i \geq 0, R_i > 0 \tag{3}$$

Agent i terminal cost

$$\ell_{i,T}(x_{i,T}) = (x_{i,T} - r_{i,T})^\intercal Q_{i,T} (x_{i,T} - r_{i,T}), \quad Q_{i,T} \geq 0 \tag{4}$$

Global system instantaneous and terminal costs

$$\ell(x_t, u_t) = \sum_{i=1}^{N} \ell_i(x_{i,t}, u_{i,t}), \quad \ell_T(x_T) = \sum_{i=1}^{N} \ell_{i,T}(x_{i,T}). \tag{5}$$

Problem Formulation

Problem 1

Choose a control trajectory $u : [\![0, T-1]\!] \to \mathbb{R}^{m_{tot}}$ *to minimize*

$$J(u) = \mathbb{E} \sum_{t=0}^{T-1} \ell(x_t, u_t) + \mathbb{E}\ell_T(x_T) \tag{6}$$

subject to the dynamics (2) *and the equality constraint*

$$\mathbb{1}_{m_{\text{tot}}}^\intercal u_t = c_t, \qquad \forall t \in [\![0, T-1]\!]. \tag{7}$$

Reminder:

$$\ell(x_t, u_t) = \sum_{i=1}^{N} \ell_i(x_{i,t}, u_{i,t}) = \sum_{i=1}^{N} \left((x_{i,t} - r_{i,t})^\intercal Q_i (x_{i,t} - r_{i,t}) + u_{i,t}^\intercal R_i u_{i,t}\right)$$

- $c_t \in \mathbb{R}$: total consumption requirement at time $t \in [\![0, T-1]\!]$.
- $m_{\text{tot}} := \sum_{i=1}^{N} d_u^i$: dimension of the input vector u_t.

Main Results for Problem 1

Theorem 1

Let $P : [\![0, T]\!] \to \mathbb{R}^{n_{tot} \times n_{tot}}$ *and* $s : [\![0, T]\!] \to \mathbb{R}^{n_{tot}}$ *be the solutions of the following backward recursions*

$$P_t = Q + A^\intercal P_{t+1} A - A^\intercal P_{t+1} B \Gamma_t B^\intercal P_{t+1} A \tag{8}$$

$$s_t = \left[A^\intercal - A^\intercal P_{t+1} B \Gamma_t B^\intercal\right] s_{t+1} + A^\intercal P_{t+1} B \gamma_t - Q r_t \tag{9}$$

with the final conditions $P_T = Q_T$ *and* $s_T = -Q_T r_T$, *where*

$$\Gamma_t = \Omega_t^{-1} - \frac{\Omega_t^{-1} \mathbb{1}_{m_{\text{tot}}} \mathbb{1}_{m_{\text{tot}}}^\intercal \Omega_t^{-1}}{\mathbb{1}_{m_{\text{tot}}}^\intercal \Omega_t^{-1} \mathbb{1}_{m_{\text{tot}}}}, \qquad \gamma_t = \frac{\Omega_t^{-1} \mathbb{1}_{m_{\text{tot}}} c_t}{\mathbb{1}_{m_{\text{tot}}}^\intercal \Omega_t^{-1} \mathbb{1}_{m_{\text{tot}}}}$$

$$\Omega_t = R + B^\intercal P_{t+1} B.$$

Then the optimal control strategy for Problem 1 is given by

$$u_t = -\Gamma_t (B^\intercal P_{t+1} A x_t + B^\intercal s_{t+1}) + \gamma_t, \qquad \forall t \in [\![0, T-1]\!]. \tag{10}$$

Γ_t projects onto $\{u : \mathbb{1}_{m_{\text{tot}}}^\intercal u = 0\}$ in the Ω_t-inner-product; γ_t is the feasibility shift making $\mathbb{1}_{m_{\text{tot}}}^\intercal u_t = c_t$.

Reduces to [Sideris-Rodriguez 2010] *when* $w_t \equiv 0$ *and to standard LQR if the constraint is dropped; extends* [Krokavec 2008]–[2014] *from state- to control-equality constraints.*

Proof of Theorem 1 (outline)

Value function:

$$V_t(z) = \min_{u_t,\dots,u_{T-1}} \mathbb{E}\left[\sum_{\tau=t}^{T-1} \ell(x_\tau, u_\tau) + \ell_T(x_T) \middle| x_t = z\right]. \tag{11}$$

1. **Bellman recursion + value-function ansatz.** Postulate

   $$V_t(z) = z^\intercal P_t z + 2 s_t^\intercal z + q_t,$$

   and write the Bellman recursion

   $$V_t(z) = \min_{u:\, \mathbb{1}_{m_{\text{tot}}}^\intercal u = c_t} \left\{ |z - r_t|_Q^2 + |u|_R^2 + \mathbb{E}\, V_{t+1}(Az + Bu + w_t) \right\}.$$

2. **Inner constrained minimisation via KKT.** Plugging the ansatz reduces the inner problem to $\min_u\{|u|_{\Omega_t}^2 + 2u^\intercal f_t(z)\}$ s.t. $\mathbb{1}_{m_{\text{tot}}}^\intercal u = c_t$, with $\Omega_t = R + B^\intercal P_{t+1} B$. The KKT conditions yield

   $$u_t = -\Gamma_t \left(B^\intercal P_{t+1} A x_t + B^\intercal s_{t+1}\right) + \gamma_t.$$

3. **Identify P_t, s_t recursions by matching coefficients.** Substituting u_t back into the Bellman equation and matching the quadratic, linear, and constant terms in z recovers the backward recursions for P_t and s_t and confirms the ansatz.

Numerical Example: Parameters

System Overview

- $N = 50$ residential batteries.
- Time Horizon: $T = 24$ hours, with $\Delta t = 1$ h.
- $x_{i,t}$: State-of-Charge in kWh.
- $u_{i,t}$: Charging/discharging in kW.
- Dynamics: $x_{i,t+1} = A_i x_{i,t} + B_i u_{i,t} + w_{i,t}$ with $A_i \in [0.96, 0.99]$, $B_i = 1$, $w_{i,t} \sim \mathcal{N}(0, 1)$ and $x_{i,0}$ uniformly drawn from $[40\%, 60\%]$.
- Cost Weights: $Q_i = Q_{i,T} = 1$, $R_i = 0.01$.

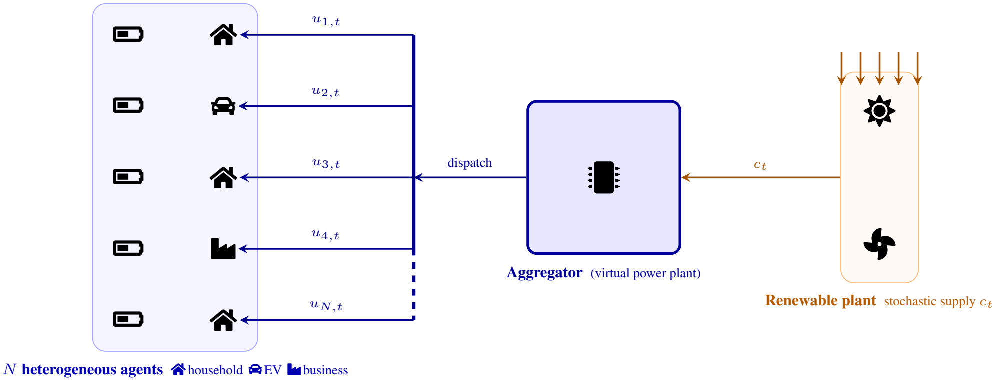
Constraint

- c_t: Total excess solar generation from real-world Montreal-East data.
- Hard Constraint: $\mathbb{1}_{m_{\text{tot}}}^\intercal u_t = c_t$.
- The batteries are equally divided into two classes α and β with distinct SoC targets:

Class	Target SoC ($r_{i,t}$)
α	80%
β	40%

Numerical Example: Problem Illustration

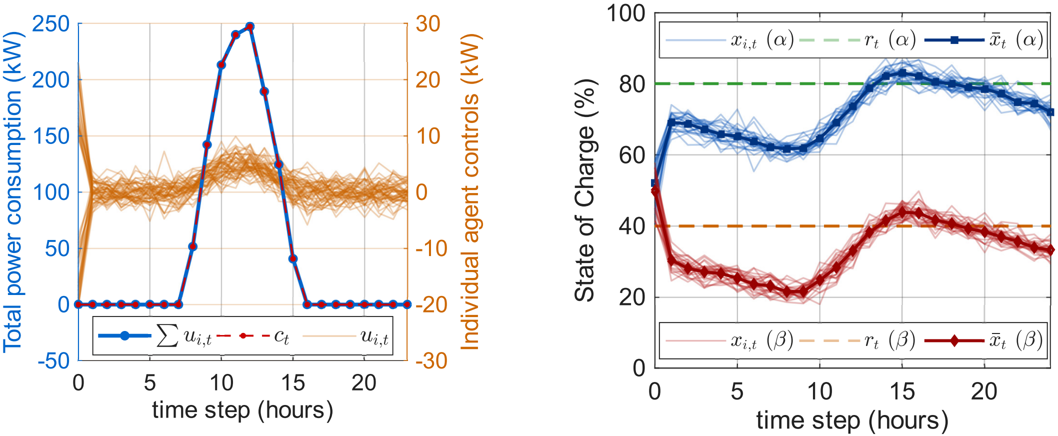
Aggregate equality constraint: $\mathbb{1}_{m_{\text{tot}}}^{\mathsf{T}} u_t = \sum_{i=1}^{N} u_{i,t} = c_t \quad \forall t$



The aggregator dispatches $(u_{i,t})_{i=1}^{N}$ so the population sum exactly matches the renewable supply c_t.

Numerical Example: Simulations

Hard Constraint activation for the full duration



(a) Constraint verification

(b) Reference tracking

Outline

Generalizations I : Intermittent Hard Constraints

Problem Formulation

Denote the set of time steps with active constraint by

$$\mathbb{S} := \{t \in [\![0, T-1]\!] \mid \sigma(t) = 1\}$$

where $\sigma : [\![0, T-1]\!] \to \{0, 1\}$ satisfies

$$\sigma(t) = \begin{cases} 1 & \text{if the hard constraint is active at time } t; \\ 0 & \text{otherwise.} \end{cases}$$

Problem 2 (Intermittent hard constraints)

Choose a control trajectory $u : [\![0, T-1]\!] \to \mathbb{R}^{m_{tot}}$ *to minimize*

$$J(u_t) = \mathbb{E} \sum_{t=0}^{T-1} \ell(x_t, u_t) + \mathbb{E}\ell_T(x_T) \tag{12}$$

subject to the dynamics (2) *and the intermittent equality constraint*

$$\mathbb{1}_{m_{tot}}^\intercal u_t = c_t, \quad \forall t \in \mathbb{S} \subseteq [\![0, T-1]\!] \tag{13}$$

where $c_t \in \mathbb{R}$ *represents the intermittent total consumption requirement at time* $t \in \mathbb{S} \subseteq [\![0, T-1]\!]$.

Proposition 1

Let $P : [\![0,T]\!] \to \mathbb{R}^{n_{tot}\times n_{tot}}$ and $s : [\![0,T]\!] \to \mathbb{R}^{n_{tot}}$ be the solutions of the backward recursions of Theorem 1 where

$$\Gamma_t = \begin{cases} \Omega_t^{-1} - \dfrac{\Omega_t^{-1}\mathbb{1}_{m_{\text{tot}}}\mathbb{1}_{m_{\text{tot}}}^\intercal\Omega_t^{-1}}{\mathbb{1}_{m_{\text{tot}}}^\intercal\Omega_t^{-1}\mathbb{1}_{m_{\text{tot}}}}, & \text{if } \sigma(t) = 1 \\ \Omega_t^{-1}, & \text{if } \sigma(t) = 0 \end{cases} \qquad \gamma_t = \begin{cases} \dfrac{\Omega_t^{-1}\mathbb{1}_{m_{\text{tot}}}c_t}{\mathbb{1}_{m_{\text{tot}}}^\intercal\Omega_t^{-1}\mathbb{1}_{m_{\text{tot}}}}, & \text{if } \sigma(t) = 1 \\ 0_{m_{\text{tot}}\times 1}, & \text{if } \sigma(t) = 0 \end{cases}$$

$$\Omega_t = R + B^\intercal P_{t+1} B.$$

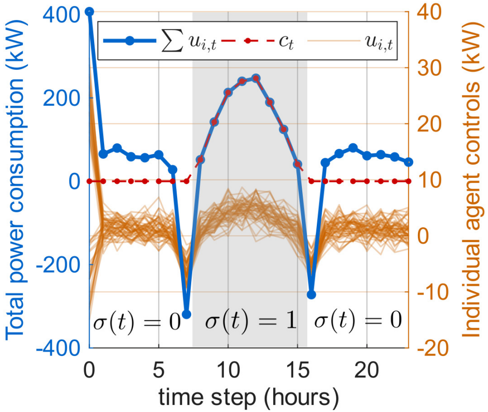
Then the stochastic optimal control strategy for Problem 2 is given by

$$u_t = -\Gamma_t(B^\intercal P_{t+1}Ax_t + B^\intercal s_{t+1}) + \gamma_t\ , \qquad \forall t \in [\![0, T-1]\!]. \tag{14}$$

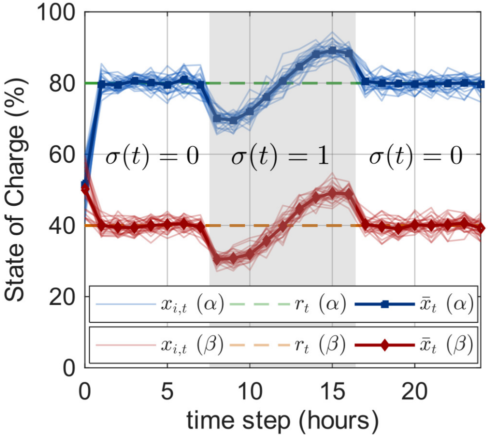
Generalization I: Intermittent Hard Constraints

Numerical Simulation

Hard constraint activation only when excess power is available $(\sigma(t) = 1)$



(a) Constraint verification



(b) Reference tracking

Global system instantaneous cost at time t

$$\ell^s(x_t, u_t) := \ell(x_t, u_t) + \eta(1 - \sigma(t))(\mathbb{1}_{m_{\text{tot}}}^\intercal u_t - c_t)^2, \tag{15}$$

and at terminal time T, $\ell_T^s(x_T) = \ell_T(x_T)$, where $\eta \geq 0$ is a scalar penalty weight.

Problem 3 (Intermittent soft and hard constraints)

Choose a control trajectory $u : [\![0, T-1]\!] \to \mathbb{R}^{m_{tot}}$ *to minimize*

$$J(u_t) = \mathbb{E} \sum_{t=0}^{T-1} \ell^s(x_t, u_t) + \mathbb{E}\ell_T^s(x_T) \tag{16}$$

while respecting the following constraint

$$\mathbb{1}_{m_{\text{tot}}}^\intercal u_t = c_t, \quad \forall t \in \mathbb{S} \subseteq [\![0, T-1]\!] \tag{17}$$

where $c_t \in \mathbb{R}$ *represents the intermittent total consumption requirement at time* $t \in \mathbb{S} \subseteq [\![0, T-1]\!]$.

Proposition 2

Let previous assumptions hold and let $P : [\![0, T]\!] \to \mathbb{R}^{n_{tot} \times n_{tot}}$ *and* $s : [\![0, T]\!] \to \mathbb{R}^{n_{tot}}$ *be the solutions of the backward recursions of Theorem 1 where*

$$\Gamma_t = \begin{cases} \Omega_t^{-1} - \dfrac{\Omega_t^{-1} \mathbb{1}_{m_{\text{tot}}} \mathbb{1}_{m_{\text{tot}}}^\intercal \Omega_t^{-1}}{\mathbb{1}_{m_{\text{tot}}}^\intercal \Omega_t^{-1} \mathbb{1}_{m_{\text{tot}}}}, & \text{if } \sigma(t) = 1 \\ \Pi_t^{-1}, & \text{if } \sigma(t) = 0 \end{cases} \qquad \gamma_t = \begin{cases} \dfrac{\Omega_t^{-1} \mathbb{1}_{m_{\text{tot}}} c_t}{\mathbb{1}_{m_{\text{tot}}}^\intercal \Omega_t^{-1} \mathbb{1}_{m_{\text{tot}}}}, & \text{if } \sigma(t) = 1 \\ \eta c_t \Pi_t^{-1} \mathbb{1}_{m_{\text{tot}}}, & \text{if } \sigma(t) = 0 \end{cases}$$

$$\Omega_t = R + B^\intercal P_{t+1} B, \qquad \Pi_t = R + B^\intercal P_{t+1} B + \eta \mathbb{1}_{m_{\text{tot}}} \mathbb{1}_{m_{\text{tot}}}^\intercal.$$

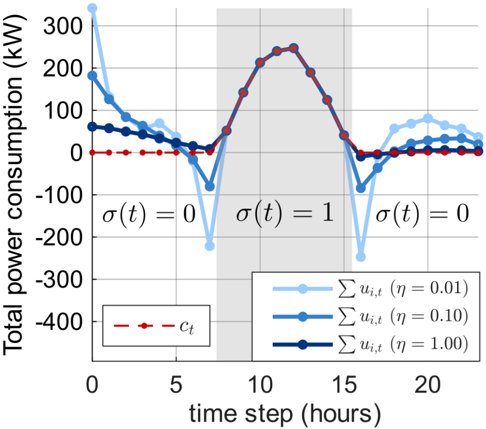
Then the optimal control strategy for Problem 3 is given by

$$u_t = -\Gamma_t (B^\intercal P_{t+1} A x_t + B^\intercal s_{t+1}) + \gamma_t \,, \qquad \forall t \in [\![0, T-1]\!]. \tag{18}$$

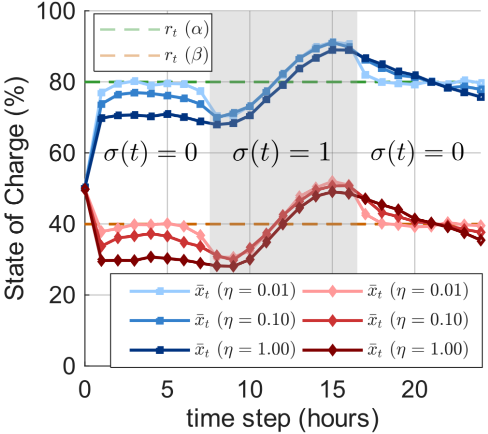
Generalizations II: Intermittent Soft and Hard Constraints

Numerical Simulation (Scenario 1)

Soft constraint applied when hard constraint is inactive $(\sigma(t) = 0)$



(a) Constraint verification



(b) Reference tracking

Generalizations II: Intermittent Soft and Hard Constraints

Numerical Example (Scenario 2): Peak-shaving Setup

System overview

- Dynamics: $x_{i,t+1} = A_i x_{i,t} + B_i u_{i,t} + w_{i,t}$, $A_i \in [0.98, 0.99]$, $B_i = 0.25\,\text{MWh/MW}$, $w_{i,t} \sim \mathcal{N}(0,3)$, initial SoC $x_{i,0} = 70\%$.
- $N = 800$ energy-storage clusters.
- Horizon $T = 96$ intervals (24 h, $\Delta t = 15$ min).
- State-of-charge in MWh (capacity 100 MWh); control $u_{i,t}$ in MW.
- Cost weights $Q_i = Q_{i,T} = 1$, $R_i = 0.01$.

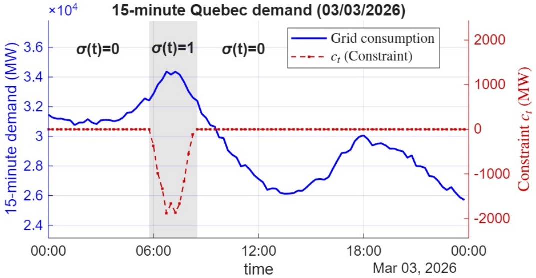
Reference

Class	Target SoC ($r_{i,t}$)
All agents	70% (70 MWh)

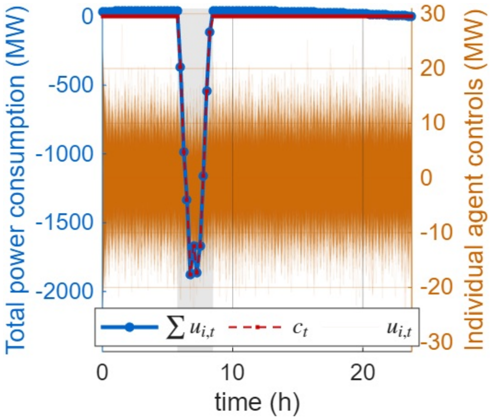
Demand-response constraint

$$\mathbb{1}_{m_{\text{tot}}}^{\mathsf{T}} u_t = c_t, \quad \forall\, t \in \mathbb{S}$$

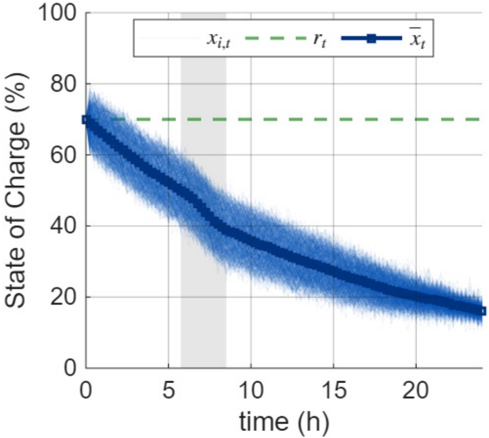
- Peak-shaving window $\mathbb{S}$: 5:45 AM to 8:30 AM.
- c_t: total discharge needed to flatten Quebec grid demand below 32,500 MW.
- Soft penalty weight $\eta = 10$.



Soft constraint applied when hard constraint is inactive ($\sigma(t) = 0$)



(a) Constraint verification

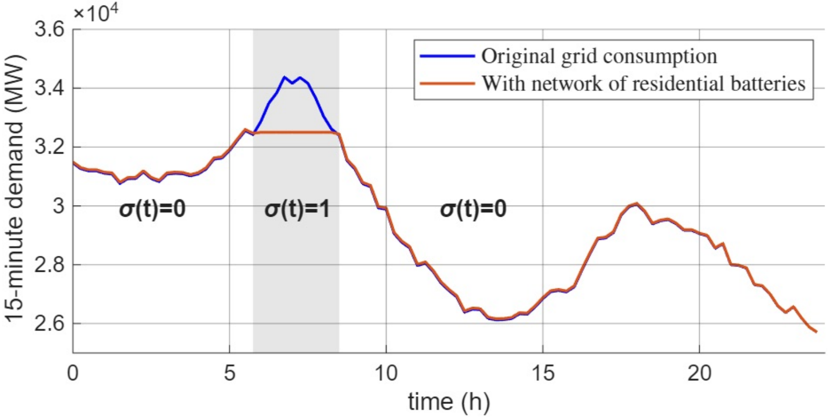


(b) Reference tracking

Generalizations II: Intermittent Soft and Hard Constraints

Numerical Simulation (Scenario 2): Grid Consumption Impact

15-minute Quebec demand — before vs. after coordinated dispatch



Conclusion and Future Work

Summary of Contributions

- Optimal control with hard constraint: Closed-form optimal gain via projection
- The Riccati structure for intermittent-hard equality constraints
- Computational cost reduction (compared to Quadratic Programming).
  $\mathcal{O}(T^3 N^3 (2n+m)^3) \to \mathcal{O}\left(TN^3\left(n^3+m^3+n^2m+nm^2\right)\right)$

Future Work

- Decentralized control with hard constraints (e.g. Mean Field Games with constraints).
- Variations of constraints (e.g. interval constraints, network-coupled constraints, random contraints).
- Data-driven coordinations

How to Coordinate Users? Related Works from My Group

- **Centralized collaboration with hard constraints** (this talk, TA11, 11:45–12:10)
  "Discrete-time linear quadratic stochastic control with equality-constrained inputs: Application to energy demand response"
  — Léo Seugnet

- **Decentralized coordination via mean field games (MFG)** (MB5, 16:45–17:10)
  "Price-Coordinated Mean Field Games with State Augmentation for Decentralized Battery Charging"
  — Nour Al Dandachly

- **Data-driven decentralized coordination via MFG** (TA5, 10:30–10:55)
  "Data-Driven Network LQG Mean Field Games with Heterogeneous Populations via Integral Reinforcement Learning"
  — Jean Zhu (also: Franz-Frédérick Acclassato)

- **Coordination with guaranteed privacy**
  — Mohadese Jadidi

Available: `https://shuanggaoee.github.io/`

Supported by FRQNT and NSERC

Thank you!

Proof of Theorem 1 (full derivation)

The cost of Problem 1 can be written as follows

$$J(u_t) = \mathbb{E}\sum_{t=0}^{T-1}\left[|x_t - r_t|_Q^2 + |u_t|_R^2\right] + \mathbb{E}|x_T - r_T|_{Q_T}^2$$

Let $V_t(z)$ be the optimal cost-to-go (or the value function) at t starting from $x_t = z$, defined as

$$V_t(z) = \min_{u_t,\dots,u_{T-1}} \mathbb{E}\left[\sum_{\tau=t}^{T-1}\ell(x_\tau, u_\tau) + \ell_T(x_T)\middle| x_t = z\right]. \tag{19}$$

We proceed by backward induction, assuming the value function at time t is of form

$$V_t(z) = z^\intercal P_t z + 2s_t^\intercal z + q_t \tag{20}$$

The Bellman recursion is as follows

$$V_t(z) = \min_u \left\{|z - r_t|_Q^2 + |u|_R^2 + \mathbb{E}\left[V_{t+1}(Az + u + w_t)\right]\right\}$$

Proof of Theorem 1 (full derivation)

subject to the constraint $\mathbb{1}_{m_{\text{tot}}}^\intercal u_t = c_t$. Assuming the previous form of the value function (20), the Bellman recursion can be written as

$$V_t(z) = |z - r_t|_Q^2 + |z|_{A^\intercal P_{t+1} A}^2 + 2 s_{t+1}^\intercal A z + \operatorname{tr}(W P_{t+1}) \\ + q_{t+1} + \min_u \left\{ |u|_{\Omega_t}^2 + 2 u^\intercal f_t(z) \right\}$$

using $\mathbb{E}(w_{t+1}^\intercal P_{t+1} w_{t+1}) = \operatorname{tr}(W P_{t+1})$, where $\Omega_t := (R + B^\intercal P_{t+1} B)$ and $f_t(z) = \left(B^\intercal P_{t+1} A z + B^\intercal s_{t+1}\right)$. To respect the constraint $\mathbb{1}_{m_{\text{tot}}}^\intercal u_t = c_t$, we introduce the following Lagrangian function

$$\mathcal{L}(u, \lambda) = u^\intercal \Omega_t u + 2 u^\intercal f_t(z) + \lambda(\mathbb{1}_{m_{\text{tot}}}^\intercal u - c_t)$$

The KKT conditions give $2\Omega_t u + 2 f_t(z) + \lambda \mathbb{1}_{m_{\text{tot}}} = 0$, which implies $u = -\Omega_t^{-1}(f_t(z) + \frac{\lambda}{2}\mathbb{1}_{m_{\text{tot}}})$. Substituting this into the constraint $\mathbb{1}_{m_{\text{tot}}}^\intercal u_t = c_t$ yields the Lagrange multiplier term

$$\frac{\lambda}{2} = -\frac{\mathbb{1}_{m_{\text{tot}}}^\intercal \Omega_t^{-1} f_t(z) + c_t}{\mathbb{1}_{m_{\text{tot}}}^\intercal \Omega_t^{-1} \mathbb{1}_{m_{\text{tot}}}}$$

Proof of Theorem 1 (full derivation)

Plugging this back into the expression for u yields the optimal action $u_t^* = -\Gamma_t(B^\intercal P_{t+1}Ax_t + B^\intercal s_{t+1}) + \gamma_t$ where

$$\Gamma_t = \Omega_t^{-1} - \frac{\Omega_t^{-1}\mathbb{1}_{m_{\text{tot}}}\mathbb{1}_{m_{\text{tot}}}^\intercal\Omega_t^{-1}}{\mathbb{1}_{m_{\text{tot}}}^\intercal\Omega_t^{-1}\mathbb{1}_{m_{\text{tot}}}}, \qquad \gamma_t = \frac{\Omega_t^{-1}\mathbb{1}_{m_{\text{tot}}}c_t}{\mathbb{1}_{m_{\text{tot}}}^\intercal\Omega_t^{-1}\mathbb{1}_{m_{\text{tot}}}}.$$

Finally, by substituting the optimal control u_t^* back into the Bellman equation and by substituting the expression for $f_t(z)$, we have

$$\begin{aligned} V_t(z) = z^\intercal &\Big[Q + A^\intercal P_{t+1}A + A^\intercal P_{t+1}B\Gamma_t B^\intercal P_{t+1}A\Big]z \\ &+ 2\Big[\left(A^\intercal - A^\intercal P_{t+1}B\Gamma_t B^\intercal\right)s_{t+1} + A^\intercal P_{t+1}B\gamma_t - Qr_t\Big]^\intercal z \\ &+ q_{t+1} + r_t^\intercal Qr_t + \mathrm{Tr}(WP_{t+1}) + \gamma^\intercal\Omega_t\gamma_t + 2s_{t+1}^\intercal B\gamma_t - s_{t+1}^\intercal B\Gamma_t B^\intercal s_{t+1}. \end{aligned}$$

We recognise the form $V_t(z) = z^\intercal P_t z + 2s_t^\intercal z + q_t$ which confirms our previous assumption on the form of V_t. The identification gives us

$$\begin{aligned} P_t &= Q + A^\intercal P_{t+1}A - A^\intercal P_{t+1}B\Gamma_t B^\intercal P_{t+1}A \\ s_t &= \left[A^\intercal - A^\intercal P_{t+1}B\Gamma_t B^\intercal\right]s_{t+1} + A^\intercal P_{t+1}B\gamma_t - Qr_t. \end{aligned}$$

■

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